THE NEW JERSEY
ITALIAN AND ITALIAN AMERICAN HERITAGE COMMISSION

Fibonacci and His Impact on Art and Architecture

Grade Level: 9-12

Time Required: Several 40 minute periods

Materials Needed:

Internet access, graph paper, poster board, interactive white board – optional, ruler, compass; portrait copies of the *Mona Lisa* and *The Vitruvian Man* by Leonardo Da Vinci, *Holy Family* by Michelangelo, *Crucifixion* by Raffaello (these can be web-based and either printed out for students or projected onto interactive white board).

Objectives:

Students will be able to:

1. identify Fibonacci as one of the central influences to the widespread use of the Hindo-Arabic numerical system and of mathematical formulas used in western education and commercial applications.
2. briefly describe Fibonacci's life and accomplishments.
3. develop connections between Fibonacci's Sequence, The Golden Rectangle, The Golden Triangle, The Golden Star, and various pieces of art by famous Italian artists of the Renaissance.
4. analyze why artists use the Golden Ratio to help develop their works of art.

Standards:

NCSS History Standards

EXPECTATION 1.1.3. - How culture influences the ways in which human groups solve the problems of daily living.

Common Core Social ELA and History and Social Studies Standards

CCSS.ELA-Literacy.RH11-12.7 – Integrate and evaluate multiple sources of information presented in diverse formats and media (visually, quantitatively, and verbally) in order to address a question or solve a problem.

CCSS.ELA-Literacy.WHST.11-12.2a – Introduce a topic and organize complex ideas, concepts and information so that each new element builds on that which precedes it to create a unified whole, include formatting (e.g. headings), graphics (e.g. figures, tables) and multimedia when useful to aiding comprehension.

CCSS.ELA-Literacy.WHST. 11-12.7 – Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem, narrow or broaden the inquiry when appropriate, synthesize multiple sources on the subject demonstrating understanding of the subject under investigation.

Common Core Mathematics Standards

CCSS.Math.Content.HSN-CN.B.5 Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.

CCSS.Math.Content.HSG-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects

CCSS.Math.Content.HSG-MG.A.3 Apply geometric methods to solve design problems

CCSS.Math.Content.HSG.GPE.B.4 Use coordinates to prove simple geometric theorems algebraically.

CCSS.Math.Content.HSG-GMD.B.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

CCSS.Math.Content.HSS-CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations

NJ Core Curriculum Content Standards for Math

4.2.12.E	Measuring Geometric Objects – strand 1
4.3.12.A	Patterns – strand 1
4.5.A	Problem Solving – strands 1-5
4.5.C	Connections – strands 1-6

NJ Core Curriculum Content Standards for World History and Global Studies

6.2.12.D.2.a Determine the factors that led to the Renaissance and the impact on the arts.

6.2.12.D.2.c Justify how innovations from Asian and Islamic civilizations, as well as from ancient Greek and Roman culture, laid the foundation for the Renaissance.

Procedures:

1. Optional Previous Night's Homework: Choose one of the following:

    a. The students should briefly research the Middle Ages (6^{th} to the 16^{th} Centuries / about 500 C.E. to about 1500 C.E.) with a focus on the history of the Roman numerical system. Roman Numerals were the primary numerical system of Europe during that time.

    b. The students should research a brief background on Italian Renaissance artists - Leonardo, Michelangelo, and Raphael.

2. As a follow up to the Previous Night's Homework, students should discuss their findings in small groups and read the Historical Background component for Leonardo Pisano Bigollo (Fibonacci).

3. From the Historical Background, the students should be able to map the progression of numerical calculations of the early 1200s as introduced to Europe by Leonardo Pisano Bigollo, also known in later years as "Leonardo Fibonacci."

4. The students should solve Fibonacci's Sequence of numbers by illustrating the number pattern from the rabbit "word problem" referenced in the Historical Background.

    "A certain man put a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits will be produced in a year, beginning with a single pair, if in every month each pair bears a new pair which becomes productive from the second month on?"

5. The answer to this problem creates a sequence of numbers. Students should identify and understand the calculation to achieve the sequence. This pattern is later called the Fibonacci Sequence.

    a. 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144…

6. The students should draw connections (through research) of how Fibonacci's number sequence is also prevalent in geometric terms.

    a. The student should experiment with Fibonacci numbers and how they connect to squares of a geometric pattern. This is called the **Golden Rectangle / Golden Ratio**.

i. The ratio for a Golden Rectangle is $\frac{1+\sqrt{5}}{2}$, or approximately 1.618.

Students can create true Golden Rectangles using graph paper, a ruler, and a compass. They can then divide this first rectangle, creating new ones.

1. Students and/or the teacher can consult the following website for assistance: http://jwilson.coe.uga.edu/emt669/student.folders/may.leanne/leanne's%20page/golden.ratio/golden.ratio.html

ii. Students can also approximate the Golden Rectangle using graph paper. Using a poster board, graph paper, white board, or interactive white board, the teacher should illustrate how the Fibonacci numbers indicate how to multiply a simple 1x1 square.

1. 1x1, 2x1, 3x2, 5x3, 8x5, 13x8, 21x13, 34x21…
2. As they construct rectangles, have students calculate the ratios of the sides:
    a. 1x1=1; 2x1=2; 3x2=1.5; 5x3=1.667; etc.
    b. Notice how, as the rectangles get larger, the ratio gets closer to the Golden Ratio.
        i. Students can calculate the ratios of large rectangles to prove that the Golden Ratio is the limit of the Fibonacci rectangles.
    c. For a visual of this, see: http://math.rice.edu/~lanius/Geom/building.html

iii. The students and/or teacher may wish to refer to the following website or others similar to provide a visual of the Golden Rectangle. http://jwilson.coe.uga.edu/EMT668/EMAT6680.2000/Obara/Emat6690/Golden%20Ratio/golden.html

http://www.mathopenref.com/rectanglegolden.html

http://www.miniwebtool.com/golden-rectangle-calculator/

b. Once the above task has been completed, the students may explore how the Golden Rectangle can then be converted into **The Golden Triangle**.

i. By dividing the Golden Rectangle into two triangles (create a line across the diagonal of the rectangle) the Golden Triangle can be created.

c. To create a **Golden Star**, Fibonacci's numbers 2, 3, 5, 8 are used as lengths of lines that connect to make a star or pentagram.

d. For further information or visual description of the **Golden Rectangle**, **Golden Triangle** or **Golden Star (pentagram)**, the teacher may wish to refer to the website below:

The Golden Ratio in Art and Architecture by Samuel Obara,
The University of Georgia

http://jwilson.coe.uga.edu/EMT668/EMAT6680.2000/Obara/Emat6690/Golden%20Ratio/golden.html

7. The students will use the following information to explain that the Golden Rectangle, the Golden Triangle and the Golden Star have been used and can be mapped within famous pieces of art from Italian Renaissance Artists.

a. As a point of reference, the Renaissance time period occurred during the end of the Middle Ages. It is sometimes referred to as a cultural era that connected the Middle Ages and the Modern Era.

i. The Middle Ages – occurred from the 6^{th} to the 16^{th} Centuries

ii. The Late Middle Ages – occurred from the 14^{th} to the 16^{th} Centuries (1300-1500)

iii. The Renaissance – occurred from 14^{th} to the 17^{th} Centuries

b. The students should refer to their research findings from homework on the Italian Renaissance Artists - Leonardo, Michelangelo, and Raffaello or conduct research during class time. Some brief information is provided below. For more in-depth information, students may wish to complete further internet research.

i. **Leonardo Da Vinci** - (1452 – 1519) His most famous works of art are his portraits of *Mona Lisa* and *The Last Supper*. He was also a scientist and blended his knowledge of the world and his talent as an artist to provide society with insight into the human anatomy and plans for future inventions of a tank and helicopter.

ii. **Michelangelo Buonarroti** – (1475 – 1564) He was a skilled and influential painter and sculptor. He took years to complete the paintings that cover the ceiling of the Sistine Chapel. Among the numerous

sculptures he completed, he is most known for his creations of *David* and the *Pietà*.

iii. **Raffaello Sanzio da Urbino** – (1483 – 1520) He has been known to have revolutionized painting during this time period due to the techniques he used. He created cartoons that can be seen on the tapestries of the Sistine Chapel. He gave credit to both Michelangelo and Leonardo as those who had inspired his own painting.

c. Obtain electronic or printed versions of the following pieces of art using internet resources. Depending upon time allotted for the class activities, the teacher may direct students to use only one, a few or all of the following.

i. *The Vetruvian Man* by Leonardo (circa 1492)

ii. *Mona Lisa* by Leonardo (1503 to 1507)

iii. *Holy Family* by Michelangelo (circa 1504-5)

iv. *Mond Crucifixion* by Raffaello (circa 1502-3)

d. Using these pieces of art, the students with the help of the teacher should consider the elements of each painting to map the Golden Rectangle, Golden Star or Golden Triangle within.

i. ***The Vitruvian Man*** – Has three rectangle sections: head, body and legs.

ii. ***Mona Lisa*** - Fibonacci's sequence creates a Golden Rectangle that begins from the top of the lady's head covering her face, shoulders, chest and down to her right wrist and across to her upper left arm.

1. For further information on Fibonacci Sequence in Art, you may want to refer to internet resources such as: The Fibonacci Series – Applications http://library.thinkquest.org/27890/applications6.html

iii. ***Holy Family*** – The elements (figures) of the portrait are positioned in a way that fit within a five point star, Golden Star or pentagram. The top point is positioned at the top of Joseph's head. Mary's head sits inside the top triangle of the star. The side points, left and right stretch across the painting toward the figures behind the family on the left and right. The center of the star encompasses Mary's body. The bottom left and right points of the star reach just below Mary's foot on the left and her knee on the right.

iv. ***Mond Crucifixion*** – The positioning of the elements of this portrait display the Golden Triangle. The top point of the triangle touches the top of Jesus' head on the crucifix. The bottom right corner of the triangle is at the foot of Mary Magdalene. The bottom left corner of the triangle is at the foot of St. Jerome. Standing outside of the triangle, on the left is Jesus' mother Mary and on the right is St. John.

Homework:

Students should use internet resources to develop a brief explanation outlining why many artists, including the four from the above lesson, use the Golden Ratio to help create their artwork.

Extension:

Have the students create an original work of art which includes elements that can fit a Golden Rectangle, Golden Star or Golden Triangle.

Historical Background

(**Optional**: Teacher may wish to reproduce the Historical Background for students to read or edit to create a shorter version for students)

The twelfth century, also known as the 1100s or the High Middle Ages, was a lively period in European history, notably for the rise of universities, and the contributions of Italian maritime republics like Genoa, Pisa, and Amalfi.

Leonardo Pisano Bigollo, "Fibonacci," has been referred to as the most advanced and talented mathematician of the Middle Ages. He is credited for inspiring the wide-spread use of the Hindo-Arabic number system throughout Europe. He was believed to have been born sometime between 1170 A.D. and 1175 A.D. in the Republic of Pisa, now a part of modern day Italy. Some historians believe the last name "Bigollo" was used as a nickname and may mean "traveler" or "good-for-nothing." It has been suggested by historians that perhaps some people of Bigollo's day believed this meaning suited him well. He was indeed a world traveler with his father, (Guglielmo Bonacci, an important trade ambassador), but also immersed himself in the study of arithmetic, which many considered to have no practical value. The name "Fibonacci" was not given to him until centuries after his death by Guillaume Libri. The nickname was formed from his father's name and the Latin root "filius Bonacci" meaning "son of Bonacci." Fibonacci is also known as Leonardo of Pisa, Leonardo Pisano, and Leonardo Bonacci.

As a young boy, Fibonacci traveled to join his father in Bugia, North Africa, which is present day Bejaia, Algeria. His father had been appointed to serve as a public notary on behalf of the Republic of Pisa. It was there that Fibonacci was educated by the Moors, who were an Arabic society of nomadic people. As a citizen of Latin speaking society, he was familiar with using the Roman numerical system which used letters that correlated to specific values. Through his education and many encounters with merchants in Bugia, Fibonacci was introduced to the "Hindo-Arabic" system of numerals. This system is based on nine digits, 1 – 9, zero and a decimal system whereby introducing a place value system. Fibonacci soon realized the value of this new system. It had fewer numerical symbols and made the calculation process more efficient and accurate than using the Roman numeral system.

Upon his return to Pisa, Fibonacci shared the concept of this new Arabic or "Indian" numerical system with society by authoring *Liber abbaci* in 1202 as Leonardo Pisano. *Liber abbaci* can be translated as "Book of Calculations" or "abacus." In this text, he provided a detailed explanation of algebraic theory. Merchants and other members of society, who frequently used accounting, also recognized the advantages of the Hindo-Arabic numerical system. In his book, Fibonacci posed mathematical questions, what students today might consider word problems. One of which inquires, *"How many pairs of rabbits will be produced in a year, beginning with a single pair, if in every month each pair bears a new pair which becomes productive from the second month on?"* The answer to this problem creates a sequence of numbers which is later called the

Fibonacci Series. The sequence is unending. The key to this sequence is adding the two previous numbers to achieve the next. In his book, Fibonacci explains in depth the answer to this question. In summary, beginning with just one pair in the first month and using the formula stated above, by the end of the twelfth month, there would be 377 pairs of rabbits. It has not been confirmed whether Fibonacci invented these word problems or if they were questions he encountered during his studies abroad and included as part of his book. This first book also introduced lattice multiplication, place value, and Egyptian fractions. Many of these concepts are now a part of modern day mathematical lessons for young students. This book was later revised by Fibonacci following his meeting with the Holy Roman Emperor. Fibonacci developed his second text in 1220 titled *Practica geometriae*. The title of this book may not be difficult to translate, *The Practice of Geometry*. This book provided an understanding of practical theorems and proofs for surveyors.

Fibonacci's work drew the attention of scholars of the Holy Roman Emperor Frederick II. They in turn convinced Frederick II that he should meet with Fibonacci. During this meeting, Frederick's scholars developed many mathematical challenges for Fibonacci to solve. This prompted Fibonacci to write his third book, *Flos,* in 1225 which means the "Flower of Mathematics." It was within this book that Fibonacci provided detailed explanation of the mathematical questions he was challenged to solve. Fredrick II and his scholars were impressed with Fibonacci's solutions and his knowledge of mathematics. They praised him as "*the serious and learned Master Leonardo Bigollo…"* for sharing his knowledge with citizens and providing guidance to the Republic of Pisa on matters of accounting.

Fibonacci's final book in 1225, *Liber quadratorum*, was considered his most impressive, yet not most famous, endeavor. This book was the Book of Squares and explores Pythagorean triples. Fibonacci did write other texts, *Di minor guise and Elements,* but because of the time era, additional copies were handwritten and very few in number. Many copies have been lost over the course of time.

Fibonacci is believed to have died approximately 1240 A.D. His contributions to the world of mathematics has spanned for almost 800 years. His journals and mathematical texts provide insight to help modern societies understand more about life during the Middle Ages, important merchandise, ports for import and export, monetary conversions, cities that minted money, etc. Streets in both Pisa and Florence have been named in his honor. A statue of Fibonacci is located near the Cathedral in Pisa and also in Florence.

Background Resources:

Russell, Deb. "A Short Biography of Leonardo Pisano Fibonacci." About.com Mathematics. 2012, <http://math.about.com/od/mathematicains/a/fibonacci.htm?p=1>

Anderson, Matt; Frazier, Jeffrey; Popendorf, Kris. “The Fibonacci Series: Biographies.” ThinkQuest. 1999. <http://library.thinkquest.org/27890/biographies1.html>

Hordadam, A.F. “Eight Hundred Years Young.” The Australian Mathematics Teacher 31 (1975). Kimberling, Clark, Evansville University, Indiana. <http://faculty.evansville.edu/ck6/bstud/fibo.html >

O’Connor, JJ; Robertson, EF. “Leonardo Pisano Fibonacci.” MacTutor History of Mathematics. October 1998. http://www-history.mcs.st-andrews.ac.uk/Biographies/Fibonacci.html

Knotts, Dr. Ron. “Who was Fibonacci?” September 28, 2009. http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibBio.html

Famous Artists of Italy. http://library.thinkquest.org/2838/artgal.htm

Renaissance. http://en.wikipedia.org/wiki/Renaissance

The Mond Crucifixion. The National Gallery. Circa 1502 -3. http://www.nationalgallery.org.uk/paintings/raphael-the-mond-crucifixion

Holy Family. Art and the Bible. Artbible.info. 2005-2012. http://www.artbible.info/art/large/508.html

Harnes, Brenda. *The Vetruvian Man*. Fine Art Touch. http://www.finearttouch.com/Da_Vinci_s_Vitruvian_Man.html